Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/22**

Paper 2 **May/June 2015**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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1 A particle P is moving in a circle of radius 0.25 m. At time t seconds, its velocity is $(2t^2 - 4t + 3)\,\mathrm{m\,s^{-1}}$. At a particular time T seconds, where $T > 0$, the magnitude of the transverse component of the acceleration of P is $6\,\mathrm{m\,s^{-2}}$. Find the magnitude of the radial component of the acceleration of P at this instant. [4]

2 A particle P moves on a straight line AOB in simple harmonic motion. The centre of the motion is O, and P is instantaneously at rest at A and B. The point C is on the line AOB, between A and O, and $CO = 10\,\mathrm{m}$. When P is at C, the magnitude of its acceleration is $0.625\,\mathrm{m\,s^{-2}}$ and it is moving towards O with speed $6\,\mathrm{m\,s^{-1}}$. Find

(i) the period of the motion, in terms of π, [2]

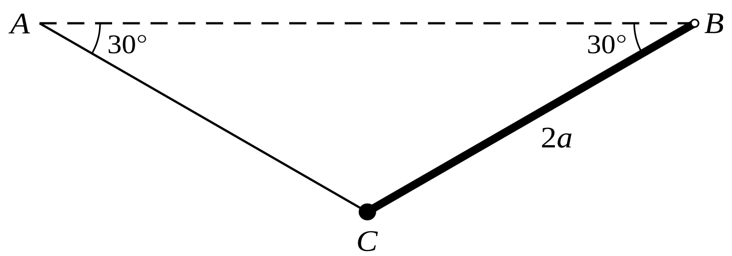
(ii) the amplitude of the motion. [2]

The point M is the mid-point of OB. Find the time that P takes to travel directly from C to M. [4]

3 A particle P, of mass m, is placed at the highest point of a fixed solid smooth sphere with centre O and radius a. The particle P is given a horizontal speed u and it moves in part of a vertical circle, with centre O, on the surface of the sphere. When OP makes an angle θ with the upward vertical, and P is still in contact with the surface of the sphere, the speed of P is v and the reaction of the sphere on P has magnitude R. Show that $R = mg(3\cos\theta - 2) - \dfrac{mu^2}{a}$. [5]

The particle loses contact with the sphere at the instant when $v = 2u$. Find u in terms of a and g. [4]

4



A uniform rod BC of length $2a$ and weight W is hinged to a fixed point at B. A particle of weight $3W$ is attached to the rod at C. The system is held in equilibrium by a light elastic string of natural length $\frac{3}{5}a$ in the same vertical plane as the rod. One end of the elastic string is attached to the rod at C and the other end is attached to a fixed point A which is at the same horizontal level as B. The rod and the string each make an angle of 30° with the horizontal (see diagram). Find

(i) the modulus of elasticity of the string, [5]

(ii) the magnitude and direction of the force acting on the rod at B. [5]

5 Three uniform small smooth spheres A, B and C have equal radii and masses $3m$, $2m$ and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between A and B is e and the coefficient of restitution between B and C is e'. Sphere A is projected directly towards B with speed u. Show that, after the collision between B and C, the speed of C is $\frac{2}{5}u(1 + e)(1 + e')$ and find the corresponding speed of B. [7]

After this collision between B and C it is found that each of the three spheres has the same momentum. Find the values of e and e'. [5]

6 The independent random variables X and Y have distributions with the same variance σ^2. Random samples of N observations of X and 10 observations of Y are taken, and the results are summarised by

$$\Sigma x = 5, \qquad \Sigma x^2 = 11, \qquad \Sigma y = 10, \qquad \Sigma y^2 = 160.$$

These data give a pooled estimate of 12 for σ^2. Find N. [4]

7 A random sample of 8 sunflower plants is taken from the large number grown by a gardener, and the heights of the plants are measured. A 95% confidence interval for the population mean, μ metres, is calculated from the sample data as $1.17 < \mu < 2.03$. Given that the height of a sunflower plant is denoted by x metres, find the values of Σx and Σx^2 for this sample of 8 plants. [7]

8 **(a)** For a random sample of ten pairs of values of x and y taken from a bivariate distribution, the equations of the regression lines of y on x and of x on y are, respectively,

$$y = 0.38x + 1.41 \qquad \text{and} \qquad x = 0.96y + 7.47.$$

(i) Find the value of the product moment correlation coefficient for this sample. [2]

(ii) Using a 5% significance level, test whether there is positive correlation between the variables. [4]

(b) For a random sample of n pairs of values of u and v taken from another bivariate distribution, the value of the product moment correlation coefficient is 0.507. Using a test at the 5% significance level, there is evidence of non-zero correlation between the variables. Find the least possible value of n. [2]

9 Cotton cloth is sold from long rolls of cloth. The number of flaws on a randomly chosen piece of cloth of length a metres has a Poisson distribution with mean $0.8a$. The random variable X is the length of cloth, in metres, between two successive flaws.

(i) Explain why, for $x \geqslant 0$, $\mathrm{P}(X > x) = \mathrm{e}^{-0.8x}$. [2]

(ii) Find the probability that there is at least one flaw in a 4 metre length of cloth. [2]

(iii) Find

(a) the distribution function of X, [1]

(b) the probability density function of X, [2]

(c) the interquartile range of X. [4]

10 Young children at a primary school are learning to throw a ball as far as they can. The distance thrown at the beginning of the school year and the distance thrown at the end of the same school year are recorded for each child. The distance thrown, in metres, at the beginning of the year is denoted by x; the distance thrown, in metres, at the end of the year is denoted by y. For a random sample of 10 children, the results are shown in the following table.

Child	A	B	C	D	E	F	G	H	I	J
x	5.2	4.1	3.7	5.4	7.6	6.1	3.2	4.0	3.5	8.0
y	6.2	4.8	5.0	5.6	7.7	7.0	4.0	4.5	3.6	8.5

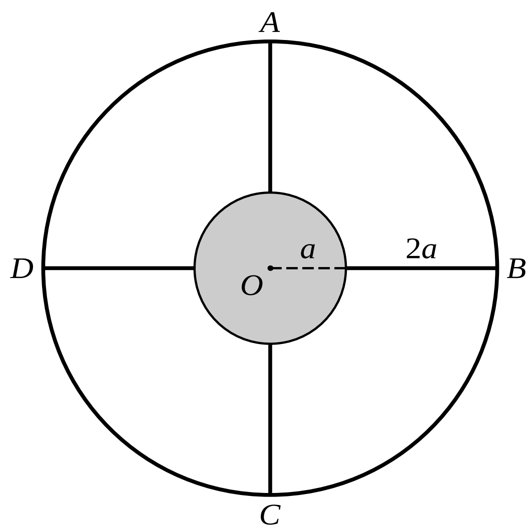
$[\Sigma x = 50.8, \quad \Sigma x^2 = 284.16, \quad \Sigma y = 56.9, \quad \Sigma y^2 = 347.59, \quad \Sigma xy = 313.28.]$

A particular child threw the ball a distance of 7.0 metres at the beginning of the year, but he could not throw at the end of the year because he had broken his arm. By finding the equation of an appropriate regression line, estimate the distance this child would have thrown at the end of the year. [5]

The teacher suspects that, on average, the distance thrown by a child increases between the two throws by more than 0.4 metres. Stating suitable hypotheses and assuming a normal distribution, test the teacher's suspicion at the 5% significance level. [8]

11 Answer only **one** of the following two alternatives.

EITHER



A uniform disc, with centre O and radius a, is surrounded by a uniform concentric ring with radius $3a$. The ring is rigidly attached to the rim of the disc by four symmetrically positioned uniform rods, each of mass $\frac{3}{2}m$ and length $2a$. The disc and the ring each have mass $2m$. The rods meet the ring at the points A, B, C and D. The disc, the ring and the rods are all in the same plane (see diagram). Show that the moment of inertia of this object about an axis through O perpendicular to the plane of the object is $45ma^2$. [4]

Find the moment of inertia of the object about an axis l through A in the plane of the object and tangential to the ring. [3]

A particle of mass $3m$ is now attached to the object at C. The object, including the additional particle, is suspended from the point A and hangs in equilibrium. It is free to rotate about the axis l. The centre of the disc is given a horizontal speed u. When, in the subsequent motion, the object comes to instantaneous rest, C is below the level of A and AC makes an angle $\sin^{-1}\left(\frac{1}{4}\right)$ with the horizontal. Find u in terms of a and g. [7]

OR

Each of 200 identically biased dice is thrown repeatedly until an even number is obtained. The number of throws, x, needed is recorded and the results are summarised in the following table.

x	1	2	3	4	5	6	$\geqslant 7$
Frequency	126	43	22	3	5	1	0

State a type of distribution that could be used to fit the data given in the table above. [1]

Fit a distribution of this type in which the probability of throwing an even number for each die is 0.6 and carry out a goodness of fit test at the 5% significance level. [8]

For each of these dice, it is known that the probability of obtaining a 6 when it is thrown is 0.25. Ten of these dice are each thrown 5 times. Find the probability that at least one 6 is obtained on exactly 4 of the 10 dice. [5]

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